

Emergence of long memory in stock volatility from a modified Mike-Farmer model

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PACS 89.65.Gh – Economics; econophysics, financial markets, business and management

PACS 89.75.Da – Systems obeying scaling laws

PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion

Abstract. - The Mike-Farmer (MF) model was constructed empirically based on the continuous double auction mechanism in an order-driven market, which can successfully reproduce the cubic law of returns and the diffusive behavior of stock prices at the transaction level. However, the volatility (defined by absolute return) in the MF model does not show sound long memory. We propose a modified version of the MF model by including a new ingredient, that is, long memory in the aggressiveness (quantified by the relative prices) of incoming orders, which is an important stylized fact identified by analyzing the order flows of 23 liquid Chinese stocks. Long memory emerges in the volatility synthesized from the modified MF model with the DFA scaling exponent close to 0.76, and the cubic law of returns and the diffusive behavior of prices are also produced at the same time. We also find that the long memory of order signs has no impact on the long memory property of volatility, and the memory effect of order aggressiveness has little impact on the diffusiveness of stock prices.

Introduction. – The continuous double auction mechanism is adopted in the electronic trading systems in many stock markets worldwide. In particular, most emerging stock markets are order-driven markets. In a pure order-driven market, there are no market makers or specialists, and market participants submit and cancel orders, which may result in transactions based on price-time priority. Different from quote-driven markets where market makers are liquidity providers, the same trader in an order-driven market can act as either a liquidity taker or a liquidity provider depending on the aggressiveness of her submitted orders. The behaviors of market makers are very complicated, since they have the obligation to maintain the liquidity of stocks and in the meanwhile want to maximize their profits. It is thus natural to argue that it is easier to construct microscopic models for order-driven markets than for quote-driven markets in order to understand the macroscopic regularities of stock markets from a microscopic angle of view.

Indeed, a lot of efforts have been made to construct order-driven models [1], which can be dated back to the 1960's [2].

In order to check if the model captures some basic aspects of the underlying mechanisms governing the evolution of stock prices, one usually investigates the statistical properties of the mock stocks, such as the distribution and autocorrelation of returns and the long memory in volatility. Deviations from these well-established stylized facts allow us to improve the models and gain a better understanding of the underlying microscopic mechanisms. For instance, the DFA scaling exponent of price fluctuations is found to be significantly less than the empirical value in the Bak-Paczuski-Shubik model [3] and in the Maslov model [4], leading to new order-driven models [5–8].

Recently, Mike and Farmer have proposed an empirical behavioral model, which is based on the statistical properties of order placement and cancellation extracted from ultrahigh-frequency stock data [9]. To the best of our knowledge, the Mike-Farmer model (or MF model for short) is the only *empirical* model, which outperforms other order-driven models and is adaptive for further improvement. The MF model can reproduce several important stylized facts: The returns are distributed according to the cubic law, the DFA scaling exponent

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of returns is close to 0.5, and the spreads and lifetimes of orders have power-law tails. However, the DFA scaling exponent of the volatility is also found to be $H_v \approx 0.6$, which is much less than the empirical value of $H_v \approx 0.8$ [9]. In this work, we propose a modified version of the MF model, which is able to produce very realistic strong persistence in the volatility without destruction of other stylized facts.

The volatility clustering phenomenon, as well as other important stylized facts, can be observed in many other microscopic market models. In the econophysics literature, physicists model stock markets as a complex system with interacting agents and different physics scenarios lead to different types of models [10], such as percolation models [11–16], spin models [17–22], minority games [23–30], majority games [31–33], and the $\$$ -game [34], to list a few. There is also a long list of stock market models in the economics literature [35]. In contrast with models where the agents (or traders) are homogenous, most of economic models assume that the traders have bounded rationality and heterogeneous beliefs [36, 37]. Traders can thus be classified into two types: fundamentalists and chartists. The fundamentalists believe that the asset price is solely determined by economic fundamentals and they buy (or sell) when the price is lower (or higher) than the fundamental price. On the contrary, chartists are trend followers and try to predict future price movement according to diverse techniques. Many theoretical and computational oriented models have been proposed [38–46].

Mike-Farmer model and its modification. – The MF model contains two main parts, order placement and cancellation. In order to submit an order, one needs to decide its direction (buy or sell), price and size. In the MF model, the size of any order is fixed to one. The sign of orders presents strong long memory, with $H_s \approx 0.8$ [47]. Therefore, order signs can be generated from fractional Brownian motions with DFA scaling exponent H_s . The price of an incoming order can be characterized by the relative price x , which is the logarithmic distance of the order price to the same best price:

$$x(t) = \begin{cases} \ln \pi(t) - \ln \pi_b(t-1), & \text{buy orders} \\ \ln \pi_a(t-1) - \ln \pi(t), & \text{sell orders} \end{cases}, \quad (1)$$

where $\pi(t)$ is the order price at time t , and $\pi_b(t-1)$ and $\pi_a(t-1)$ are the best bid and best ask at time $t-1$, respectively. The relative prices in the MF model are generated from a Student distribution whose degrees of freedom α_x and scaling parameter σ_x are determined empirically using real stock data. Mike and Farmer also proposed a model for order cancellation combining three factors: the position of an order in the order book, the imbalance of buy and sell orders in the book, and the total number of orders in the book.

With these findings in hand, our simulations of the MF model can be described as follows. Before the evolution of prices, we generate an array of relative prices $\{x(t) : t = 1, 2, \dots, T\}$, drawn from the Student distribution with $\alpha_x = 1.3$ and $\sigma_x = 0.0024$, and an array of signs $\{s(t) : t = 1, 2, \dots, T\}$ according to a fractional Brownian motion with $H_s = 0.75$. At each simulation step t , an order is generated, whose relative price

and direction are $x(t)$ and $s(t)$, respectively. If $x(t)$ is not less than the spread, the order is an effective market order, resulting in an immediate execution with a limit order waiting at the opposite best price. Otherwise, the incoming order is an effective limit order, which is stored in the queue of the limit order book. Then we scan the standing orders to check if any of them can be canceled, following exactly the same process in the MF model. We simulate $T = 2 \times 10^5$ steps in each round. The stock prices are recorded and we analyze the last 4×10^4 returns in each round.

The distribution of returns in the MF model has been studied in detail and we reproduced the cubic law [48]. We now perform a detrended fluctuation analysis (DFA) [49, 50] on the return r and the volatility $v = |r|$ to estimate the DFA scaling exponents. The results are shown in Fig. 1. Excellent power-law dependence of the detrended fluctuation function $F(\ell)$ with respect to the timescale ℓ is observed for the two quantities in the scaling range $8 \leq \ell < 7000$. The DFA scaling exponents are $H_r = 0.55$ for the returns and $H_v = 0.58$ for the volatility, respectively. These indexes are merely a little greater than 0.5, which means that there is no long memory or very weak memory in the returns and the volatility. To obtain a solid picture, we repeated the simulations of the MF model 20 times and performed DFA on the returns and the volatility. We find that H_r varies in the range $[0.54, 0.58]$ with the average $\overline{H}_r = 0.57 \pm 0.01$ for the returns, and H_v varies in the range $[0.56, 0.62]$ with the average $\overline{H}_v = 0.59 \pm 0.01$ for the volatility. This analysis confirms the results of Mike and Farmer [9]. It is well accepted in mainstream Finance that there is no memory in returns [51], consistent with the weak-form market efficiency hypothesis, while the volatility possess strong persistence with the DFA scaling exponent much greater than 0.5 [52]. Therefore, the MF model captures the stylized fact that H_r of returns is close to 0.5, but fails to reproduce strong memory effect in the volatility. Obviously, certain important feature is missing in the original MF model, which calls for a further scrutiny of the real stock data and a modification of the model.

In financial markets, it is impossible for a trader to collect and digest all information that is available publicly, and it is not free to collect and process diverse information from different sources. Due to the limited processing power of human brains and finite amount of money, it is not irrational for traders to mimic the trading behaviors of others, which may lead to positive feedbacks and herding behaviors in an intermittent fashion. In other words, most traders in financial markets play a majority game. They are more willing to buy when the price rises and to sell when the price falls. This scenario is known as the information cascading mechanism [53] and it is well documented that imitation and herding cause the emergence of volatility clustering and long memory. A comprehensive taxonomy of herd behavior was synthesized by Hirshleifer and Teoh [54]. We also refer to an excellent book of Lyons for a modern treatment [55]. Following this line, a trader is very possible to submit an order that is “similar” to its preceding limit orders. In addition, the long memory in the order flow is well-known as “diagonal effect” [56]. Other than herding, there are at least two alternative

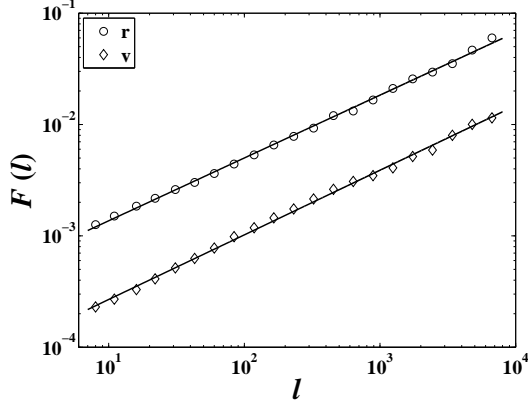


Fig. 1: Detrended fluctuation function $F(\ell)$ as a function of time lag ℓ for the returns and the volatility, respectively. The solid lines are the linear least-squares fits to the data and $H_r = 0.55 \pm 0.01$ for returns and $H_v = 0.58 \pm 0.01$ for volatility. The plot for volatility has been shifted vertically for clarity.

hypotheses for the origin of long memory in the order flow: order splitting and traders reacting similarly to the same signal [56]. Since an order is fully determined by its direction (order sign), aggressiveness (order price) and size, we expect that these variables might also have strong memory. In the MF model, the directions of incoming orders are modeled by fractional Brownian motions with $H_s \gg 0.5$, while the order size is fixed. It is thus worthwhile to check if the order aggressiveness characterized by relative prices has long memory using real ultrahigh-frequency stock data, and if the long memory in the order aggressiveness, if any, can cause the emergence of long memory in the volatility.

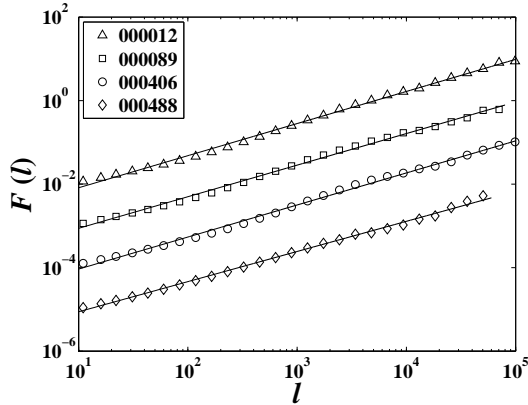


Fig. 2: Dependence of the detrended fluctuation function $F(\ell)$ with respect to the timescale ℓ for four stocks, whose stock codes are 000012, 000089, 000406 and 000488. The solid lines are the linear least-squares fits to the data and $H_x = 0.77 \pm 0.01$, 0.76 ± 0.01 , 0.77 ± 0.01 , and 0.72 ± 0.01 , respectively. The plots for stocks 000089, 000406 and 000488 have been shifted vertically for clarity.

In order to study the memory effect of order aggressiveness, we utilize a nice database of 23 liquid stocks listed on the Shenzhen Stock Exchange in the whole year 2003 [57]. The

database contains detailed information of the incoming order flow, such as order direction and size, limit price, time, best bid, best ask, transaction volume, and so on. We focus on the relative prices of orders submitted during the continuous double auction. Figure 2 illustrates the dependence of the detrended fluctuation functions $F(\ell)$ with respect to the timescale ℓ for four randomly chosen stocks. Sound power-law scaling relations are observed in the scaling ranges spanning four orders of magnitude. The DFA scaling exponents of the relative prices for the four stocks are estimated to be $H_x = 0.77 \pm 0.01$ in the scaling range $10 \leq \ell < 10^5$, 0.76 ± 0.01 in the scaling range $10 \leq \ell < 7 \times 10^4$, 0.77 ± 0.01 in the scaling range $10 \leq \ell < 10^5$, and 0.72 ± 0.01 in the scaling range $10 \leq \ell < 5 \times 10^4$, respectively. The DFA results for other stocks are quite similar. We find that H_x varies in the range $[0.72, 0.87]$ with an average $\overline{H}_x = 0.78 \pm 0.03$. It is evident that the relative price x is super-diffusive and possesses long-term dependence.

It is noteworthy to point out that the long memory temporal structure in the relative prices was also observed in the London Stock Exchange. Zovko and Farmer studied the autocorrelation function of relative prices for buy orders and sell orders of 50 stocks traded on the London Stock Exchange [58]. They found that the autocorrelation function decays as a power law with exponent $\gamma = 0.41 \pm 0.07$. It follows immediately that $H_x = 1 - \gamma/2 = 0.80 \pm 0.04$ [59]. We also performed detrended fluctuation analysis of the relative prices for buy orders and sell orders of the four stocks analyzed in Fig. 2. The exponents are 0.75 ± 0.01 , 0.81 ± 0.01 , 0.77 ± 0.01 and 0.70 ± 0.01 for buy orders and 0.77 ± 0.01 , 0.75 ± 0.01 , 0.75 ± 0.01 and 0.71 ± 0.01 for sell orders. There is no significant difference in the memory properties if one considers relative prices of orders on the same side of the book.

Based on the above empirical finding that the relative prices have long memory, we can introduce a new ingredient in the MF model. The modified MF model inherits all the ingredients of the MF model except that the relative prices are generated from a Student distribution with long memory. This can be done as follows. We generate an array of relative prices $\{x_0(t) : t = 1, 2, \dots, T\}$ from a Student distribution. Then we simulate a fractional Brownian motion with $H_x = 0.8$ and record its differences as $\{y(t) : t = 1, 2, \dots, T\}$. The sequence $\{x_0(t) : t = 1, 2, \dots, T\}$ is rearranged such that the rearranged series $\{x(t) : t = 1, 2, \dots, T\}$ has the same rank ordering as $\{y(t) : t = 1, 2, \dots, T\}$, that is, $x(t)$ should rank n in sequence $\{x(t) : t = 1, 2, \dots, T\}$ if and only if $y(t)$ ranks n in the $\{y(t) : t = 1, 2, \dots, T\}$ sequence [60, 61]. It is obvious that $x(t)$ still obeys the same Student distribution. A detrended fluctuation analysis of $x(t)$ shows that its DFA scaling exponent is very close to $H_x = 0.8$. This sequence of $x(t)$ is used as the relative prices in our modified MF model.

Numerical results. — Based on the modified MF model discussed above, we first generate the relative prices x from the Student distribution with parameters $\alpha_x = 1.3$ and $\sigma_x = 0.0024$. Then we add long memory to the time series, using $H_x \approx 0.8$. In each round, we simulate the modified MF model

2×10^5 steps with the same parameters $H_s = 0.75$, $A = 1.12$ and $B = 0.2$ and record the return time series with the length near 4×10^4 after removing the transient period. In Fig. 3, we illustrate a typical segment of the simulated returns from the modified MF model, which is compared with the return time series of a real Chinese stock (code 000012) and the original MF model. It is evident that the return time series of the modified MF model exhibits clear clustering resembling the clustering phenomenon in real data, whereas the simulated returns from the original MF model do not show clear clustering feature. This already indicates qualitatively that the volatility of the modified MF model has stronger long-term memory than that of the original MF model.

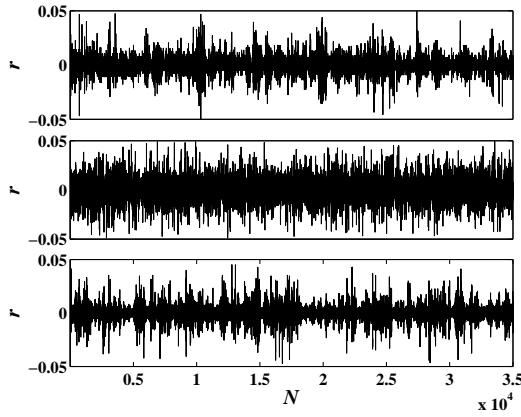


Fig. 3: Comparison of typical return time series from a real Chinese stock 000012 (upper panel), the original MF model (middle panel), and the modified MF model (lower panel).

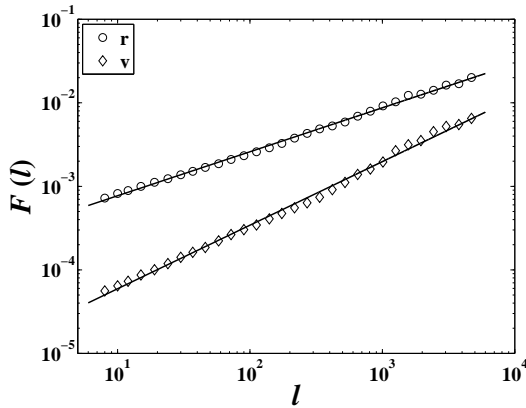


Fig. 4: Detrended fluctuation analysis of the returns r and the volatility v generated according to the modified MF model. The solid lines are the linear least-squares fits to the data and $H_r = 0.53 \pm 0.01$ for the returns and $H_v = 0.76 \pm 0.01$ for the volatility. The plot for volatility has been shifted vertically for clarity.

To quantify the strength of the memory effect in the simulated volatility, we have performed the detrended fluctuation analysis. Figure 4 shows the dependence of the detrended fluctuation $F(\ell)$ as a function of the timescale ℓ in log-log coordinates. We find that $F(\ell)$ scales as a power law against ℓ

with the scaling range spanning about three orders of magnitude. We obtain $H_v = 0.76 \pm 0.01$ in the scaling range $8 \leq \ell < 4500$, which is in excellent agreement with empirical results. We also performed a detrended fluctuation analysis on the returns. The results are also presented in Fig. 4. We find that $H_r = 0.53 \pm 0.01$ in the scaling range $8 \leq \ell < 4500$, consistent with empirical results. Comparing with Fig. 1, we conclude that the value of H_x has little impact on H_r . We repeated this process for 20 times and the results are very similar. The exponent H_v varies in the range $[0.74, 0.77]$ with an average $\overline{H}_v = 0.76 \pm 0.01$, while H_r ranges in $[0.53, 0.55]$ with an average $\overline{H}_r = 0.54 \pm 0.01$.

In order to further inspect the quantitative relation between H_x and H_v , more simulations with different values of H_x have been performed. For each fixed H_x , repeated simulations do not show much fluctuation in H_v . The results are shown in Table 1. It is found that H_v is not identical to H_x . However, H_v increases with H_x . Table 1 also confirms that H_r is close to 0.5 and independent of H_x . The relation between volatility clustering and relative prices has been detected and investigated for stocks on the London Stock Exchange [58].

Table 1: Dependence of H_v and H_r on H_x . For each value of H_x , ten repeated simulations are conducted. The scaling range is $8 \leq \ell < 4500$. The numbers in the parentheses are the standard deviations divided by 100.

H_x	0.50	0.60	0.70	0.80	0.90
H_v	0.57(1)	0.61(1)	0.67(1)	0.76(1)	0.81(2)
H_r	0.55(1)	0.55(1)	0.54(1)	0.54(1)	0.54(1)

Figure 5 shows the empirical complementary cumulative distribution $P(> v)$ of the volatility generated according to the modified MF model. We find that the volatility has a power-law tail

$$P(> v) \sim v^{-\beta}, \quad (2)$$

where β is the tail index. Using the least-squares fitting method, we obtain that $\beta = 2.99 \pm 0.02$, identical to 3. In other words, the volatility obeys the well-known cubic law [62], which is captured by the original MF model [9, 48].

Additional numerical experiments show that the cancellation process in the modified MF model is not the only one to reproduce the main stylized facts. The modified MF model with a Poissonian cancellation process gives $H_r = 0.51 \pm 0.01$, $H_v = 0.81 \pm 0.01$, and $\beta = 3.19 \pm 0.03$.

Beside efficiency and long memory of the volatility and the cubic law of the return, the price dynamics is characterized by multifractality [52]. We adopted the multifractal detrended fluctuation analysis [63] to investigate the return and volatility time series generated from the MF model, the modified MF model, and the real data as well for comparison. For a given time series, the q -th order detrended fluctuation function $F_q(s)$ scales as a power law

$$F_q(s) \sim s^{H(q)} \quad (3)$$

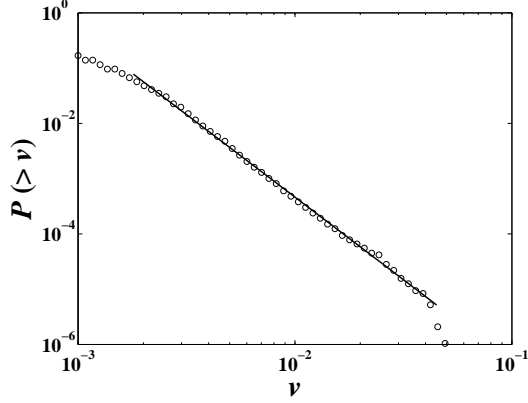


Fig. 5: Empirical complementary cumulative distribution $P(> v)$ of the volatility generated according to the modified MF model in double logarithmic coordinates. The solid line is the best power-law fit to the data with the tail index $\beta = 2.99 \pm 0.02$.

and the mass exponent $\tau(q)$ in the standard textbook structure function formalism is [63]

$$\tau(q) = qH(q) - 1. \quad (4)$$

Note that $H(q = 2)$ is the DFA scaling exponent characterizing the long memory property of the time series. The mass exponent $\tau(q)$ of each financial variable is plotted in Fig. 6 as a function of q . When $q = 0$, $\tau(0) = -1$ for each case, as predicted by Eq. (4). It is evident that all $\tau(q)$ functions are nonlinear with respect to q , which confirms the multifractal nature of return and volatility in both models and in real data. When $q < 0$, both models deviate remarkable from real data. When $q \geq 0$, both models reproduce quantitatively similar $\tau(q)$ function of the return as real data, and the $\tau(q)$ function for the volatility from the MF model deviates from that of the real data while the modified MF model captures excellently the multifractality in real data.

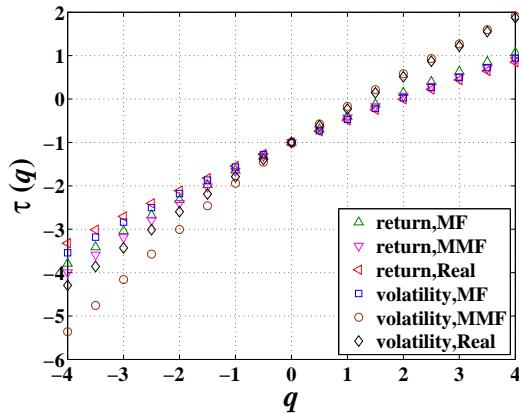


Fig. 6: Multifractal detrended fluctuation analysis of the returns r and the volatility v generated according to the MF model and the modified MF model with comparison to the multifractal nature of the real data.

We have shown that our modified MF model is able to produce long memory in the volatility while keeping the cubic

law and nonpersistence in the returns. The last but not least question is if the long memory in the relative prices alone can reproduce the long memory in the volatility when there is no memory in the order signs. To address this question, we performed extensive simulations following the MF model but with $H_s = 0.5$ and $H_x = 0.8$. We find that the $H_v = 0.78$, remaining unchanged when compared with the modified model in which $H_s = 0.75$ and $H_x = 0.8$. Moreover, the volatility is also distributed according to the cubic law. In addition, we have $H_r = 0.42$, indicating that the prices evolve in a weak sub-diffusive behavior, which is nevertheless not far from the diffusive regime with $H_r = 0.5$. We note that some stocks do show weak sub-diffusion effect [51].

Concluding remarks. – In summary, we have improved the Mike-Farmer model for order-driven markets by introducing long memory in the order aggressiveness, which is an important stylized fact identified using the ultra-high-frequency data of 23 liquid Chinese stocks traded on the Shenzhen Stock Exchange in 2003. A detrended fluctuation analysis of the relative prices x unveils that $\overline{H}_x = 0.78 \pm 0.03$. The modified MF model is able to produce long memory in the volatility with $\overline{H}_v = 0.79 \pm 0.02$, which is much greater than $\overline{H}_v = 0.59 \pm 0.01$ obtained from the original MF model. When we investigate the temporal correlation of returns, we find that $\overline{H}_r = 0.53 \pm 0.01$, indicating that the prices are diffusive. In addition, the cubic law for the return distribution holds in the modified MF model. Our modified MF model also enables us to distinguish the isolated memory effects of order directions (H_s) and aggressiveness (H_x) on the correlations in returns (H_r) and the volatility (H_v). We find that H_v is strongly dependent of H_x and irrelevant to H_s . In contrast, H_r depends strongly on H_s with little impact from H_x . We confirmed that both the MF model and the modified MF model are able to produce multifractality in the simulated prices.

The price formation process is fully determined by the dynamics of order submission and order cancelation. Intuitively, the order submission process has more important impact on the emergence of long memory in the volatility. There are four factors in the order submission process, the DFA scaling exponent H_s of order signs, the order size, the distribution $f(x)$ and the DFA scaling exponent H_x of relative prices. Our simulations show that the distribution $f(x)$ might have impact on the return distribution [48] but not the long memory in the volatility. Therefore, we figure that the long memory of order aggressiveness is a nontrivial main component of volatility clustering. To be more rigorous, order size may be an alternative component of volatility clustering. Indeed, order sizes are also long-term correlated [64–68] and there is well-established positive volume-volatility correlation [69]. However, the MF model and the modified MF model do not include order size as an ingredient. This issue could be addressed when a more realistic model is available, which is beyond the scope of the current work.

We are grateful to the anonymous referees for their invaluable

able suggestions. This work was partly supported by NSFC (70501011), Fok Ying Tong Education Foundation (101086), NCET (07-0288), and Shanghai Educational Development Foundation (2008SG29).

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